

# KS3 Maths Progress

Confidence • Fluency • Problem-solving • Progression

$\theta$

TWO

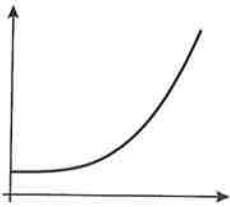
Progression Workbook

Includes videos linked  
from QR codes!



# 9.1 Direct proportion on graphs

**1 Reasoning** Joseph says, 'The graph shows one variable in direct proportion to another'. Explain why Joseph is wrong.

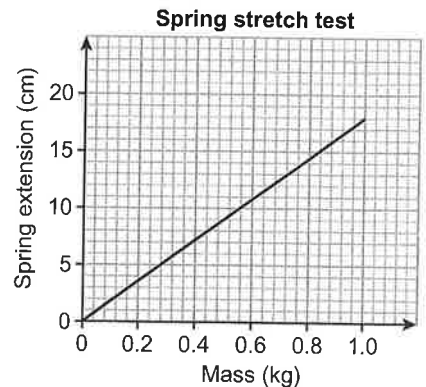


When two quantities are in direct proportion

- plotting them as a graph gives a straight line through the origin
- when one variable is zero, the other variable will also be zero
- when one variable doubles, so does the other.

**2 STEM / Reasoning** An engineer gets these results in an experiment to test a spring by applying different masses to it.

**a** In this experiment, is the spring extension in direct proportion to the mass applied? Explain how you know.



**b** Use the graph to work out the extension of the spring when these masses are applied to it.

- i** 0.2 kg .....
- ii** 0.75 kg .....
- iii** 1.5 kg .....

**Strategy hint**  
The graph doesn't go to 1.5 kg. Use values you know to work out larger values.

**c** Use the graph to work out the mass applied when the spring is extended by

- i** 0 cm .....
- ii** 9 cm .....
- iii** 27 cm .....

**3 Real** The table shows some equivalent lengths in inches and centimetres (cm).

Inches	Centimetres
2	5.1
5	12.7
10	25.4

**a** Plot a line graph for these values.

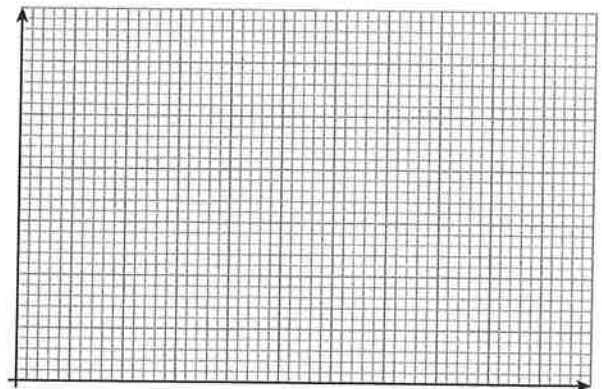
**b** Convert

**i** 4 inches to centimetres.

**ii** 20 centimetres to inches.

Centimetres (cm)

Length conversion



Inches

**c** Which is longer, 7 inches or 17 cm?

**CHECK** Tick each box as your confidence in this topic improves.



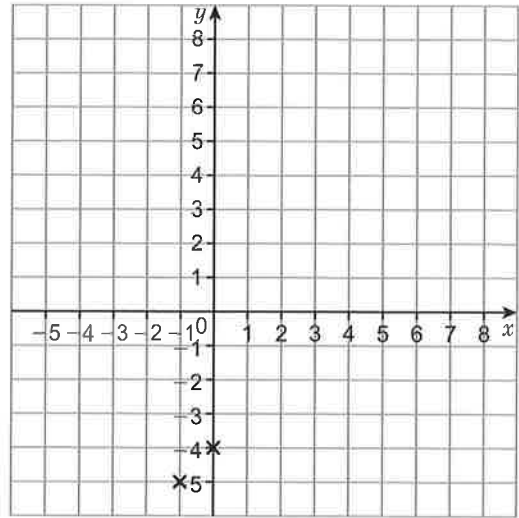
**Need extra help?** Go to page 89 and tick the box next to Q6. Then have a go at it once you've finished 9.1–9.4.

**1 a** Complete the table of values for the equation  $y = x - 4$

Guided

$x$	-1	0	1	4	8
$y$	$-1 - 4 = -5$	$0 - 4 = -4$			

- b** Plot the coordinates for  $y = x - 4$ .  
Join the points in a straight line and label the line A.
- c** How many squares does line A go up for every one square across? .....
- d** Is the gradient of line A positive or negative? .....



The steepness of the graph is called the gradient. To find the gradient, work out how many units the graph goes up for every one unit across.

**2 a** Complete this table of values for the equation  $y = -x - 1$ .

Guided

$x$	-4	-2	0	2	4
$y$		$-(-2) - 1 = 1$	$-0 - 1 = -1$		

- b** Plot the coordinates for  $y = -x - 1$  on the grid in Question 1. Join the points in a straight line and label the line B.
- c** What is the gradient of line B? .....
- d** Draw a line segment from (4, -5) to (8, 3). What is the gradient? .....
- e** Draw a line segment from (2, 3) to (8, 6). What is the gradient? .....
- f** Draw a line segment from (-2, 6) to (0, 0). What is the gradient? .....
- g** Draw a line segment from (-1, 4) to (5, 2). What is the gradient? .....

A gradient can be a fraction. Write the fraction in its simplest form. 3 up with 6 across is  $\frac{3}{6} = \frac{1}{2}$ . Check whether each gradient is positive or negative.

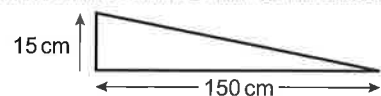
**3 Reasoning** Find the missing values.

- a** A gradient of  $\frac{1}{3}$  means ..... up for every 1 across.
- b** A gradient of  $\frac{1}{10}$  means ..... up for every ..... across.
- c** A gradient of  $\frac{2}{3}$  means ..... up for every 3 across.
- d** A gradient of  $-\frac{1}{2}$  means ..... down for every 1 across.
- e** A gradient of  $-\frac{3}{4}$  means ..... down for every ..... across.
- f** Which of these gradients is for the steepest line? .....  
Explain why. ....



You could sketch these on squared paper.

**4 Real** The diagram shows a wheelchair ramp.



- a** Work out the gradient of the ramp. ....
- b** Write the gradient in part a as a percentage. ....

Convert the fraction to a percentage.

CHECK

Tick each box as your confidence in this topic improves.



**Need extra help?** Go to pages 88 and 89 and tick the boxes next to Q1, 2 and 3. Then have a go at them once you've finished 9.1-9.4.

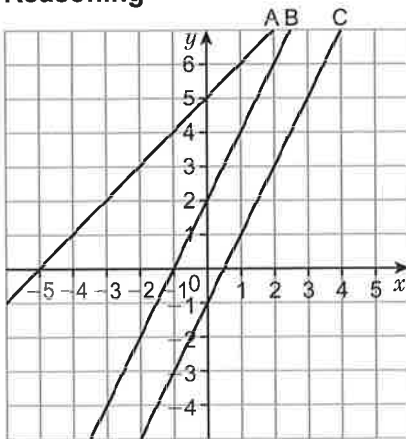
- 1 Work out the midpoint of a line segment AB, where  
 a A is (0, 3) and B is (2, 11).

- b A is (-2, -5) and B is (2, -9).

The midpoint of a line segment is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

## 2 Reasoning



### Worked example



- a Work out the gradient of line A. ....  
 b Write down the  $y$ -intercept for line A. ....  
 c Use your answers to a and b to write the equation of line A.

The  $y$ -intercept is where a line crosses the  $y$  axis.

Guided

$$y = \dots x + \dots$$

↑
↑  
 gradient       $y$ -intercept

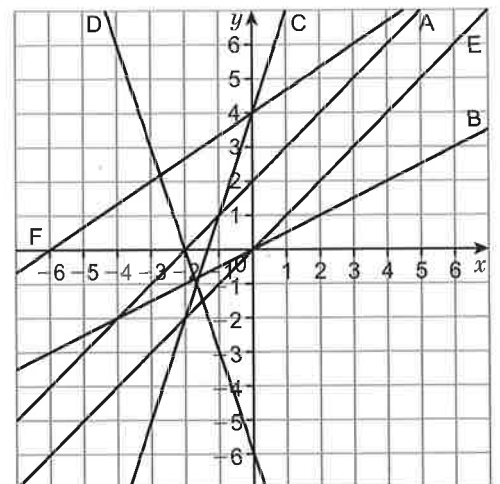
A linear equation generates a straight-line graph. The equation for a straight-line graph can be written as  $y = mx + c$  where  $m$  is the gradient and  $c$  is the  $y$ -intercept.

- d Write the equation of line  
 i B ..... ii C .....  
 e Lines B and C are parallel. What do you notice about their gradients? .....  
 f On the grid with lines A, B and C, draw and label the graph of the equations  
 i  $y = 2x - 3$     ii  $y = 2x$

## 3 a Write the equations of these lines.

- A .....  
 B .....  
 C .....  
 D .....  
 E .....  
 F .....

- b Write down the letters of any lines which are parallel. ....



### CHECK

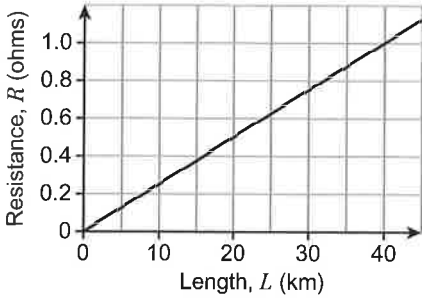
Tick each box as your confidence in this topic improves.



**Need extra help?** Go to pages 88 and 89 and tick the boxes next to Q4 and 5. Then have a go at them once you've finished 9.1–9.4.

**1 STEM** Engineers get these results when testing the resistance of different lengths of high voltage electricity cable.

Resistance of high voltage electricity cable



**a** Are resistance and length of cable in direct proportion?

Explain. ....

**b** Find the equation of the graph in the form  $y = mx + c$  .....

**c** Write a formula linking length ( $L$ ) and resistance ( $R$ ) .....

Replace  $y$  with  $R$   
and  $x$  with  $L$ .

**d** Use your formula to work out the resistance of a cable with a length of

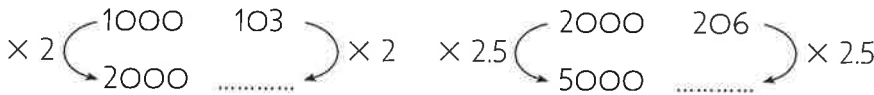
- i** 20 km .....      **ii** 30 km .....      **iii** 50 km .....


**2 STEM** The table shows the pressure and the density of seawater at different depths.

Depth (m)	1000	2000	5000
Pressure (kg/cm <sup>2</sup> )	103	206	515
Density (kg/m <sup>3</sup> )	1033	1037	1050

**a** Is the pressure in direct proportion to the depth?

Guided



  
Assume they are in direct proportion. Work out what the pressures would be. Do they match?

**b** Is the density in direct proportion to the depth?

**3 Problem-solving / Reasoning** Four weightlifters are training for a competition. The number of repetitions (reps) of their training weight per training session are

- A** 1, 6, 11, 16, 21, ...
- B** 4, 8, 16, 32, ...
- C** 3, 6, 9, 12, ...
- D** 1, 2, 3, 5, 8, ...

**a** Which of these sequences will produce a straight-line graph? .....

**b** Are any of these direct proportion relationships? Explain your answer.

CHECK

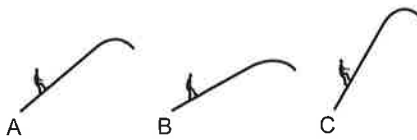
Tick each box as your confidence in this topic improves.



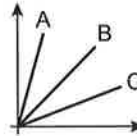
**Need extra help?** Go to page 89 and tick the box next to Q7. Then have a go at it once you've finished 9.1–9.4.

## Straight-line graphs

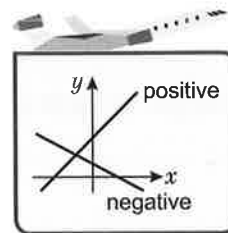
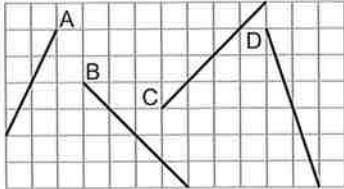
- 1 a Which hill is steepest? .....



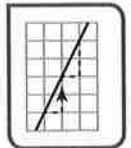
- b Which line is steepest? .....



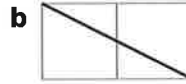
- 2 The diagram shows four lines.



- a Which lines have a positive gradient? .....
- b Choose a point on line A.  
Move your finger one square across (to the right).  
i How many squares does your finger move up to meet line A again? .....
- ii What is the gradient of line A? .....
- c Repeat part b to work out the gradient of line  
i B ..... ii C ..... iii D .....



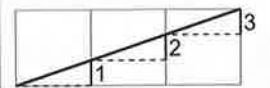
- 3 These lines have fraction gradients. Write the gradient of each line.



.....

.....

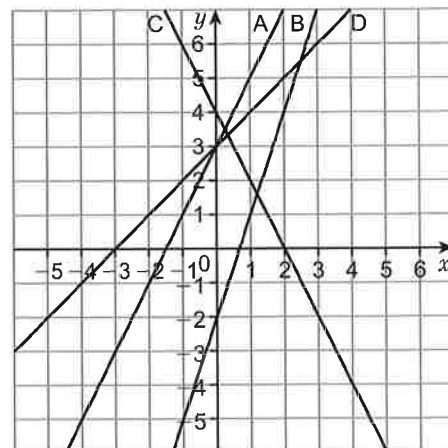
How many equal steps does it take to go up one whole square?



## Finding equations of graphs

- 4 a Work out the gradient of line A. ....
- b Write down the *y*-intercept of line A. ....
- c Complete the equation of line A.  
 $y = \dots\dots\dots x + \dots\dots\dots$   
gradient *y*-intercept
- d Complete the equation of line B.  
 $y = \dots\dots\dots x + \dots\dots\dots$
- e Write down the equations of these lines.  
i C .....  
ii D .....

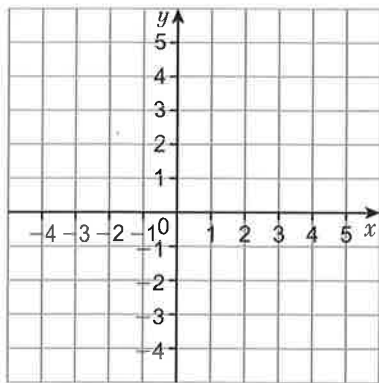
The *y*-intercept is the value where the line crosses the *y*-axis.



Decide if the gradient is positive or negative.

# Midpoints

**5 a** Draw the line segment AB with end points A (2, 1) and B (4, 5) on your grid.



Coordinates can include fractions.

**Worked example**

**b** Work out the coordinates of the midpoint, M.

Guided

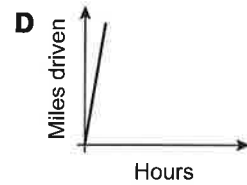
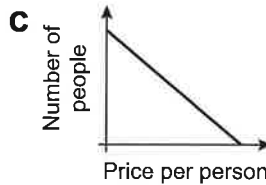
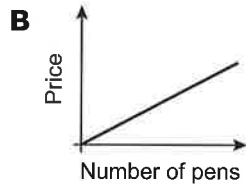
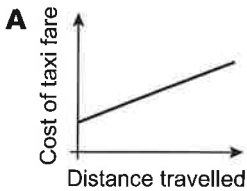
$$\begin{array}{l}
 (x, y) \\
 A \quad (2, 1) \\
 B \quad (4, 5) \\
 M \quad (\dots, \dots) \\
 \begin{array}{cc}
 \nearrow & \nwarrow \\
 \frac{2+4}{2} & \frac{1+5}{2}
 \end{array}
 \end{array}$$

**c** Work out the coordinates of the midpoints of these line segments:

- i** from C (-3, 0) to D (1, 4)
- ii** from E (-2, 1) to F (4, -1)
- iii** from G (-4, 2) to H (5, 5)

# Direct proportion

**6 Real** Which of these graphs show direct proportion?



**7** Are these quantities in direct proportion?

**a** .....

Mass of apples	Cost
1 kg	£1.60
2 kg	£3.20
3 kg	£4.80

**b** .....

Phone calls	Monthly charge
0	£10
100	£10
200	£20

**Strategy hint**

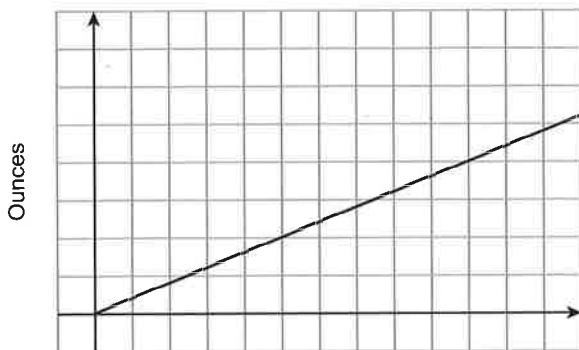
When one value is zero, is the other value zero?  
 When one value is doubled, is the other value doubled?

**1 Real** A bird flies 9 km in half an hour. How far will it fly in 40 minutes?

**2 Real / Problem-solving** It takes 6 bakers 2 hours to mix dough for 300 loaves.  
How long will it take 3 bakers to mix dough for the next 300 loaves?

Will it take 3 bakers more or less time than 6 bakers?

**3** This graph converts grams to ounces.



Grams  
1 ounce  $\approx$  25 grams

**a** Are grams and ounces in direct proportion? .....

**b** Work out the gradient of the line.

**c** Write down the equation of the line. ....

**d** 1 ounce  $\approx$  25 grams

How does the equation show this relationship?



Write in some values of ounces and grams on the graph to help.

$y = \square x + \square$

**4** Write the equations of the graph lines for these approximate conversions.

Look back at Q3.

$y$        $x$

**a** €1      £1      .....

**b** 1 inch      25 mm      .....

**c** 1 m      39 inches      .....

**d** 1 km      0.6 mile      .....

**5** Weights of lead for scuba divers' weight belts is sold by the kg.

The price of 1 kg of lead is £ $x$

Write an expression for the cost of

**a** 2 kg .....

**b** 15 kg .....



**6 STEM** A scientist measures a pair of variables.

$x$	1	2	3.5
$y$	4	8	14



If you plotted the points on a graph, would they form a straight line? Would the line go through (0, 0)?

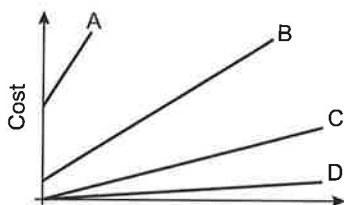
- a** Work out  $y \div x$  for each pair.
- b** Is  $y \div x$  the same each time? .....
- c** Are  $y$  and  $x$  in direct proportion? .....

**7 Problem-solving** It takes a microwave oven 40 seconds to heat 200 ml of soup. The time taken to heat soup is proportional to the volume of soup. How long does it take to heat 500 ml of soup?

**Worked example**

**8 Problem-solving / Reasoning**

- a** Match each graph A to D with calculations i to iv.
- i** The cost of  $n$  kg of aluminum cans at 50p per kg. ....
  - ii** The cost of  $n$  sweets at 22p each. ....
  - iii** The cost of hiring a canoe at £10 plus £5 for every hour of hire. ....
  - iv** The cost of a taxi fare at £3 plus £1 per km. ....



**b** Explain how you matched graph B to your answer.

**9 STEM** Various amounts of chemical  $c$  are added to an acid. The different amounts of gas  $g$ , given off in the reaction are measured in three experiments.

Work out  $g \div c$  for each pair. If they are all the same, then  $c$  and  $g$  are in direct proportion.

**A**

$c$	2.2	4.7	6
$g$	17.6	37.6	48

**B**

$c$	3.5	3.6	7.2
$g$	38.5	39.6	86.4

**C**

$c$	0.4	2.8	4.9
$g$	0.46	3.22	5.635

In which experiments are the quantities are in direct proportion? .....

**PROGRESS BAR** Colour in the progress bar as you get questions correct. Then fill in the progression chart on pages 103–105.

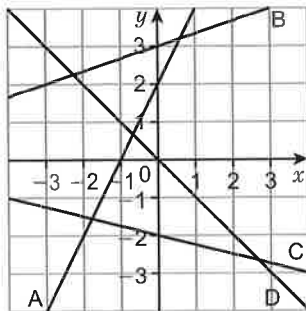
1 Three T-shirts cost £21. How much do 5 T-shirts cost?

2 Work out the midpoint of the line segment AB where

a A is (4, 2) and B is (12, 6)

b A is (-4, 0) and B is (2, -5)

3 Work out the gradient of each line.



A .....

B .....

C .....

D .....

4 Work out the equation of each line in Question 3.

A .....

B .....

C .....

D .....

5 The table shows the prices of different masses of jet fuel.

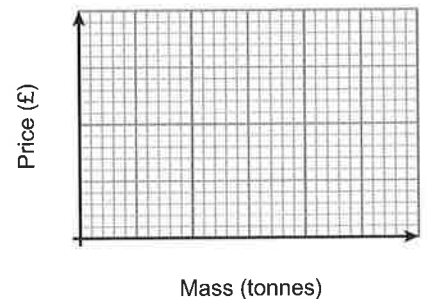
Mass (tonnes)	0.5	1.5	3
Price (£)	300	900	1800

a Plot a graph of these values.

b Are the price and mass in direct proportion? Explain.

c What is the price per tonne of jet fuel? .....

d Write an equation linking price and mass. ....



6 Are units of gas used and cost in direct proportion? Show your working.

Units used	Cost
50	£27.25
100	£29.50
200	£34

7 For each pair of values of  $x$  and  $y$ , work out if they are in direct proportion.

If they are, write an equation linking  $x$  and  $y$ .

a

$x$	3	10	14
$y$	22.5	86	119

b

$x$	2.5	4	5.5
$y$	5.25	8.4	11.55