

# KS3 Maths Progress

Confidence • Fluency • Problem-solving • Progression

$\theta$   
ONE

Progression Workbook

Includes videos linked  
from QR codes!





1 For each sequence

i write down the next three terms

ii write down the term-to-term rule.

Guided

a i  $3, 10, 17, 24, 31, \dots$

ii add 7

b i  $12, 15, 18, 21, \dots$

ii  $\dots$

c i  $50, 48, 46, 44, \dots$

ii  $\dots$

d i  $8, 8.4, 8.8, 9.2, \dots$

ii  $\dots$

e i  $10, 9.5, 9, 8.5, \dots$

ii  $\dots$

$3 + 7 = 10, 10 + 7 = 17, 17 + 7 = 24$ , so  $24 + 7 = 31$   
To get from one term to the next you 'add 7'.

A number sequence is a set of numbers that follow a rule. Each number in a sequence is called a term. The term-to-term rule tells you how to get from one term to the next in a sequence. It can use adding, subtracting, multiplying and dividing.

Worked example



2 A sequence has a first term of 5. The term-to-term rule is 'add 2'.

Guided

a Write down the first five terms of the sequence.

5,  $\dots$

The second term is  $5 + 2 = \square$

b Is the sequence finite or infinite?

A sequence that carries on for ever is infinite.  
A sequence with a fixed number of terms or a 'last term' is finite.

3 Use the first term and the term-to-term rule to generate the first five terms of each sequence.

a First term 0, term-to-term rule '+ 1.1'.  $\dots$

b First term 20, term-to-term rule '- 0.3'.  $\dots$

4 **Problem-solving** One grain of rice is placed on the first square of a chessboard, two grains on the second, four grains on the third, and so on, doubling the number on each square.

a How many grains will be on the fifth square?  $\dots$

b Which square will have 64 grains?

Strategy hint

You could draw a table like this to help.

Square	1	2	3	4	...
Grains	1	2	4	?	...

5 Work out the three missing terms in each sequence.

a  $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, \dots$

b  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

c  $\dots, 13, 15, 17, \dots$

d  $4, \dots, 14, \dots, 24, \dots$

e  $-2, \dots, -6, \dots, -10, \dots$

f  $\dots, 8.9, 8.6, 8.3, \dots$



From 13 to 15 the term-to-term rule is 'add 2'.  
 $\square + 2 = 13$

Find the difference between two of the terms, then halve it to find the term-to-term rule.

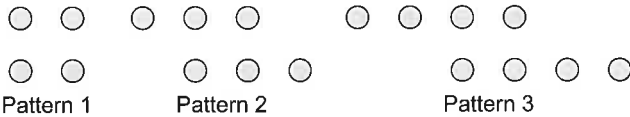
CHECK

Tick each box as your confidence in this topic improves.



Need extra help? Go to page 97 and tick the box next to Q3. Then have a go at it once you've finished 9.1–9.6.

1 Look at this sequence of patterns made from counters.



- a Draw the next pattern in the sequence.
- b Complete this table for the sequence.

Pattern number	1	2	3	4	5
Number of counters					

c Describe how the sequence grows.

You can draw the next pattern in a sequence by working out how the pattern grows.

**Worked example**



You can describe how a sequence grows by explaining how to get from one pattern to the next.

2 Harry is saving for a new hockey stick. He starts with £20. He plans to save £15 each week until he has £110.

a Complete the table to show how much money he has each week.

Week number	1	2	3	4	5
Amount of money (£)	20	35			

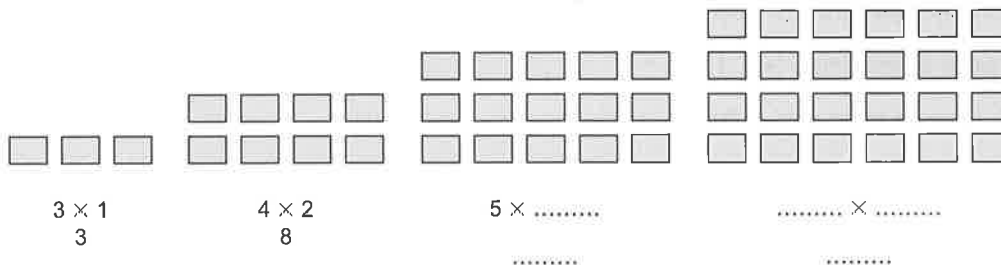
b In which week will he reach his target? .....

Harry starts with £20. The next week he adds £15.  $20 + 15 = 35$ . In week 3 he has  $35 + 15 = \square$

Continue the table up to £110. Read the week number.

Guided

3 This is a sequence of growing rectangles.



- a Complete the multiplications for the rectangles.
- b Describe in your own words how this sequence grows.
- c The 2nd rectangle is  $4 \times 2$ . What will the 8th rectangle be?

4 **Problem-solving / Finance** Anna and Bob buy a bamboo plant each.

Anna's bamboo is 25 cm high and grows 15 cm per day.  
 Bob's bamboo is 50 cm high and grows 10 cm per day.  
 Who will have the bigger bamboo after 1 week? By how much?

CHECK

Tick each box as your confidence in this topic improves.



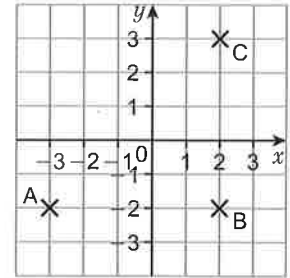
**Need extra help?** Go to page 97 and tick the box next to Q1. Then have a go at it once you've finished 9.1–9.6.

# 9.3 Coordinates

1 a Write down the coordinates of points A, B and C.

A .....  
 B .....  
 C .....

Always write the  $x$ -coordinate first and the  $y$ -coordinate second:  $(x, y)$ .

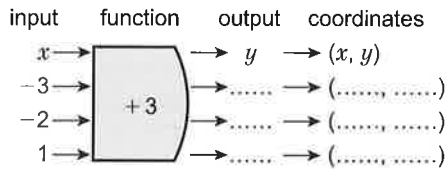


b A, B and C are three vertices of a square.

Write down the coordinates of the fourth vertex, D. ....

2 This function machine generates coordinates.

When you input an  $x$ -coordinate, it outputs the  $y$ -coordinate.

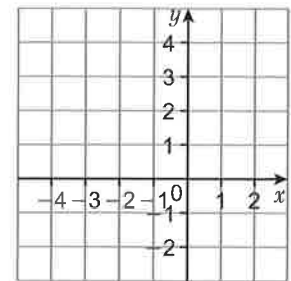


a Work out the missing  $y$ -coordinates and complete the function machine.

b Plot the coordinates from part a on the grid and join them with a straight line.

c Write down the coordinates of another point that lies on the straight line. ....

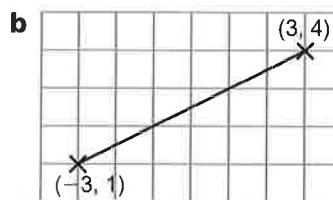
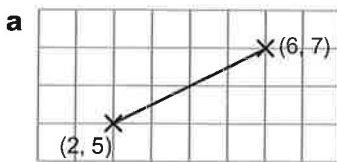
d The  $y$ -values make a sequence.  
 What is the term-to-term rule of the sequence of  $y$ -values?



e Write down the rule for the function machine using algebra.  
 Label your graph with this rule.

Your rule should start  $y = \dots$

3 Work out the midpoint of each of these line segments.



Add the two  $x$ -coordinates together and divide by 2.


Repeat for the two  $y$ -coordinates.

Guided

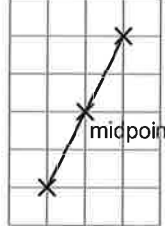
$(2 + 6) \div 2 = 4$

$(\dots + \dots) \div 2 = \dots$

midpoint =  $(4, \dots)$



The midpoint of a line segment is the point exactly in the middle.



4 Look back at Question 1. Work out the midpoint of the line segment AB.

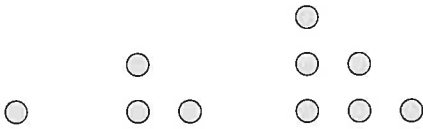
CHECK

Tick each box as your confidence in this topic improves.



Need extra help? Go to page 98 and tick the boxes next to Q6 and 7. Then have a go at them once you've finished 9.1–9.6.

1 Look at these patterns made from counters.



- a Draw the next pattern in the sequence.
- b The first three terms of the sequence are 1, 3, 6.  
Write down the next three terms. ....
- c What is the special name for the numbers in this sequence?  
.....

Look at the differences between terms.

Look at the shape of each pattern.

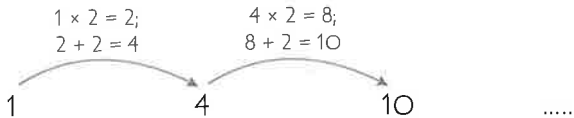
**d Reasoning** Is this an arithmetic sequence?  
Explain your answer.

An arithmetic sequence goes up or down in equal steps. For example, the sequence 4, 7, 10, 13, ..., goes up in steps of 3.

Guided

2 Write the first four terms of each sequence.

a First term 1, term-to-term rule 'multiply by 2, then add 2'.



b First term 10, term-to-term rule 'multiply by 2, then subtract 5'.

c First term 37, term-to-term rule 'subtract 3, then divide by 2'.

In part **a** the first term is 1, so write it down first.  
Be careful when the rule has two steps. Use the correct priority of operations.  $1 \times 2 + 2 = 2 + 2 = 4$

For part **c** use a function machine  
 $37 \rightarrow \boxed{-3} \rightarrow \boxed{\div 2} \rightarrow \text{next term}$   
 or brackets like this  
 $(37 - 3) \div 2 = \text{next term}$

Guided

3 Sally is training for a 10 kilometre race. She uses a treadmill every day. She runs for 30 minutes on the first day of training. Every day she adds 6 minutes to the time spent on the treadmill.

a After how many days is Sally running for 1 hour? .....

Day	1	2	3	4	5	6
Time	30	36	42			

$\xrightarrow{+6}$     $\xrightarrow{+6}$

b Do the times on the treadmill form an arithmetic sequence? Explain your answer.

4 Which of these sequences are arithmetic? For each arithmetic sequence, write down the first term and the common difference.

- a 5, 6, 8, 11, ... ..
- b 5, 10, 15, 20, ... ..
- c 5, 8, 10, 12, 15, ... ..
- d 5, 2, -1, -4, ... ..

You can describe an arithmetic sequence using the first term and the common difference (the difference between terms). For the sequence 14, 11, 8, 5, 2, ..., the first term is 14 and the common difference is -3.

**CHECK** Tick each box as your confidence in this topic improves.

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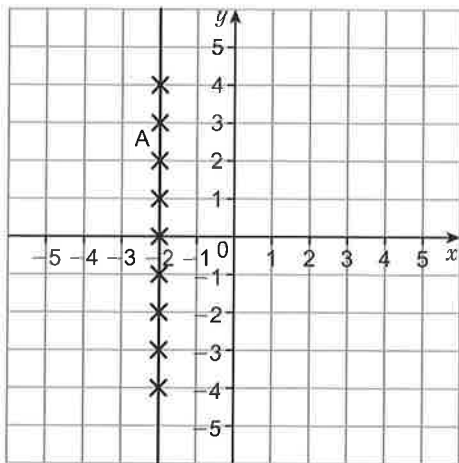
**Need extra help?** Go to page 97 and tick the box next to Q2. Then have a go at it once you've finished 9.1-9.6.

# 9.5 Straight-line graphs

- 1 a** Write down the coordinates of all the points marked with crosses on line A on the grid below.
- b** Complete this sentence: The equation for line A is  $x = \dots\dots\dots$
- c** Kelvin's rule for generating coordinates is, 'Whatever the  $x$ -coordinate is, the  $y$ -coordinate is always 2.' Circle the coordinate pairs that satisfy Kelvin's rule.  
 (0, 0) (2, 4) (4, 2) (4, 4) (2, 2) (-5, 2) (2, -5)
- d** Plot your circled points from part **c** on the grid. Join the points with a straight line and label it B. What do you notice?

When you draw a graph, it should go to the edge of the grid.

- e** Complete this sentence: The equation of line B is  $y = \dots\dots\dots$



The equation  $x = 3$  means the  $x$ -coordinate is always 3, whatever the  $y$ -coordinate is. The line is parallel to the  $y$ -axis. The equation  $y = 3$  means the  $y$ -coordinate is always 3, whatever the  $x$ -coordinate is. The line is parallel to the  $x$ -axis.

- 2 a** Complete this table of values for the equation  $y = 2x + 3$ .

$x$	0	1	2	3	4
$y$	3	5			



To find each missing  $y$ -coordinate multiply the  $x$ -value by 2, then add 3.

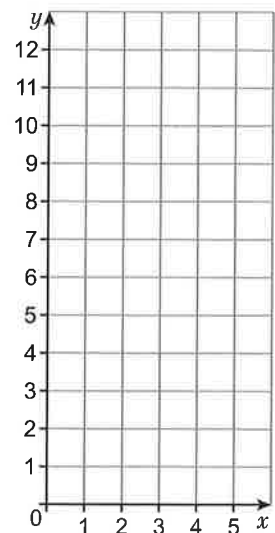
- b** Complete these coordinate pairs from the table of values.  
 (0, 3), (1, 5), (2, .....), (3, .....), (4, .....)

When  $x = 1$ ,  $y = 2 \times 1 + 3 = 5$ , giving (1, 5).

- c** Plot the coordinates on the grid on the right. Draw a straight line that goes through all the points and to the edges of the grid. Label the graph with its equation.
- d** What is the value of  $y$  when  $x = 5$ ? .....

Draw a line from 5 on the  $x$ -axis up to your graph, then across to the  $y$ -axis.

- e** What is the value of  $y$  when  $x = 3\frac{1}{2}$ ? .....



- 3 a** Complete this table of values for the function  $y = -x$ .

$x$	-3	-2	-1	0	1	2	3
$y$	3		1			-2	

- b** Use the coordinate grid in Question 1. Plot the coordinates from the table. Join them with a straight line that goes to the edges of the grid. Label the graph with its equation.

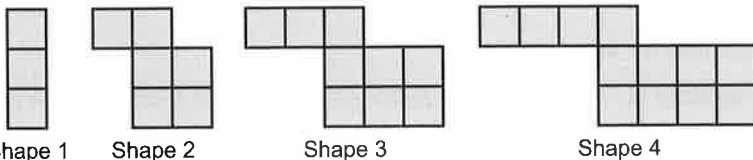
**CHECK**

Tick each box as your confidence in this topic improves.



**Need extra help?** Go to page 98 and tick the box next to Q8. Then have a go at it once you've finished 9.1–9.6.

1 Look at these shapes made from squares.



- a Draw the next shape.
- b Complete the table.

Shape number (position)	1	2	3	4	
Number of squares (term)					

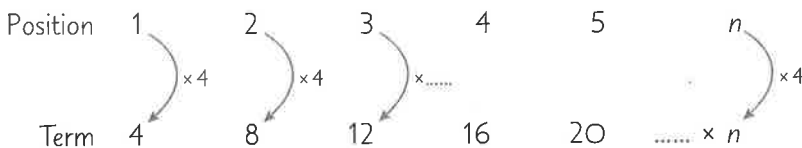
- c Write down the term-to-term rule. ....
- d How many squares are in the 6th shape? .....
- e Complete this formula to describe the position-to-term rule.  
number of squares = shape number  $\times$  .....
- f Use the position-to-term rule to find the 10th term in the sequence. ....
- g Use the position-to-term rule to find the 20th term in the sequence. ....

Each term in a sequence has a position. The 1st term is in position 1, the 2nd term is in position 2, and so on. The position-to-term rule tells you how to work out a term in a sequence when you know its position.

2 Work out the  $n$ th term of this sequence using algebra.

4, 8, 12, 16, 20, ...

Guided



$n$ th term is .....

You use algebra to write the position-to-term rule. It is called the  $n$ th term because it tells you how to work out the term at position  $n$  (any position).

3 Look at this sequence.

Position	1	2	3	4	5
Term	10	11	12	13	14

- a Write down the term-to-term rule. ....
- b Describe the position-to-term rule in words.
- c Write down the  $n$ th term using algebra. ....

**Worked example**

4 Find the  $n$ th term of the sequence 6, 12, 18, 24, ...

5 **Modelling** Keene uses a rowing machine to train for a sponsored row across the English Channel. He rows for 5 minutes on his first day, then 5 minutes more each day.

- a Continue this sequence to the 6th term. 5, 10, .....
- b What is the  $n$ th term for this sequence? .....
- c How many minutes will Keene row for on the 10th day? .....
- d Do you think Keene will be able to stick to the model for 30 days? Explain your answer.

CHECK

Tick each box as your confidence in this topic improves.

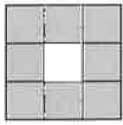


**Need extra help?** Go to page 98 and tick the boxes next to Q4 and 5. Then have a go at them once you've finished 9.1–9.6.

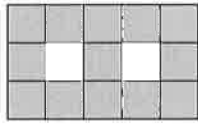


## Sequences

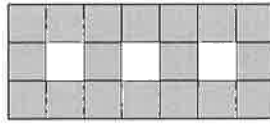
1 Here is a sequence of patterns made from grey squares.



Pattern 1



Pattern 2



Pattern 3

a Write down the terms of the number sequence.

Pattern	1	2	3
Number of grey squares (term)			

b How many grey squares are added between Patterns 1 and 2? .....

c How many grey squares are added between Patterns 2 and 3? .....

d How many grey squares will be in Pattern 4? .....

e The number sequence begins 8, 13, 18, ...  
Write down the next five terms. ....



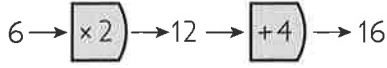
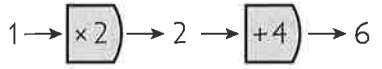
Count and write down the number of grey squares in each pattern.

This should be the same as your answer to part b because it is an arithmetic sequence.

2 Write down the first four terms of each sequence.



a First term 1, term-to-term rule 'multiply by 2, then add 4' 1, 6, 16, .....



When you input each term into the term-to-term rule, the output is the next term.

b First term 2, term-to-term rule 'subtract 1, then multiply by 3'

c First term 16, term-to-term rule 'add 8, then divide by 2'

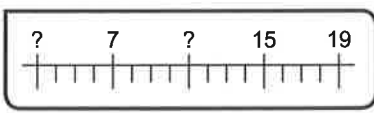
3 Work out the missing terms in each sequence.

a ....., 7, ....., 15, 19

b 34, ....., ....., 16, 10

c 2, ....., 16, ....., 30

d -6, ....., 0, ....., 6



# The $n$ th term



4 Find the position-to-term rule for each sequence.

Guided

**a**

Position	1	2	3	4	5
Term	3	4	5	6	7

Rule is .....

**b**

Position	1	2	3	4	5
Term	10	20	30	40	50

.....



You could draw a function machine with input 'position' and output 'term'. What is the rule for this function machine?

input (position)	1	2	3	4	5
output (term)	10	20	30	40	50



5 Write down the  $n$ th term for these sequences.

Use your position-to-term rule to write the 10th term and the 50th term for the sequences.

**a** 2, 4, 6, 8, ...

$n$ th term .....

10th term  $10 \times \dots = \dots$

50th term .....

**b** 10, 20, 30, 40, ...

$n$ th term .....

10th term .....

50th term .....

Guided

# Graphs



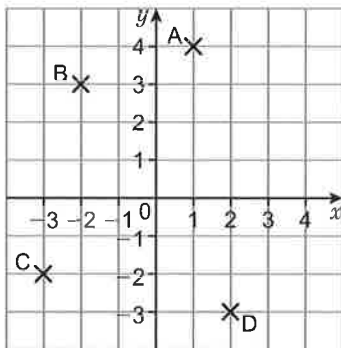
6 Write down the coordinates of each point marked with a letter.

A .....

B .....

C .....

D .....



How to read coordinates:  
 The first coordinate is where the line down reaches the  $x$ -axis (4,  $\square$ ).  
 The second coordinate is where the line across reaches the  $y$ -axis ( $\square$ , 2).

The coordinates of the point shown are (4, 2).

**Worked example**



7 Plot the points E(1, 3) and F(1, -3) on the grid and label them.

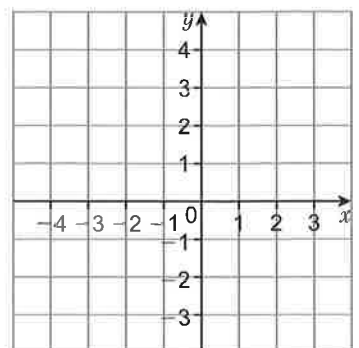


8 On the grid in Question 7, draw these lines and label each with its equation.

**a**  $y = 3$

**b**  $x = -4$

Plot some points whose  $y$ -coordinate is 3, then draw a line through them.



1 For the sequence 80, 40, 20, ..., write down

- a the first term .....      b the term-to-term rule. ....

2 **Problem-solving** Write two different ways to continue the number sequence 2, 4, ...

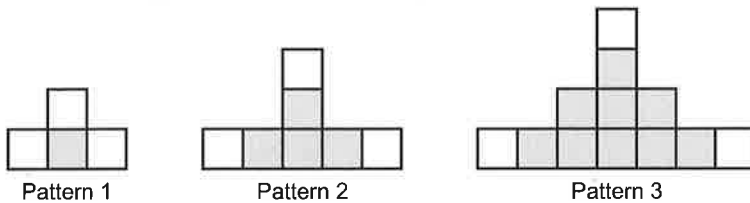
- a i 2, 4, .....  
 ii 2, 4, .....

Think of different ways to get from 2 to 4 and keep that sequence going for two or three more terms.

b Are either of your sequences in part a infinite? Explain your answer.

3 Write a sequence containing the numbers 2 and 14, with at least one term in between them. Describe the term-to-term rule that you use.

4 Look at this pattern made from squares.



- a How many grey squares will there be in the 4th pattern? .....  
 b What is the special name for the number sequence of the grey squares?  
 c How many white squares will there be in the 4th pattern? .....  
 d How many squares in total will there be in the 10th pattern?

Rearrange the grey squares of patterns 2 and 3 into square shapes.

5 Write down i the first four terms and ii the 20th term of each sequence.

- a general term  $n + 5$       b general term  $5n - 1$       c general term  $n^2 + 5$

- a i .....      ii .....  
 b i .....      ii .....  
 c i .....      ii .....



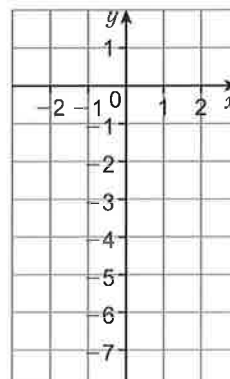
The  $n$ th term is sometimes called the general term.

6 This question is about the function  $y = 2x - 3$ .

a Complete the table with the value of  $y$  for each value of  $x$ .

$x$	-2	-1	0	1	2
$y$		-5			

- b Use the table of values to draw the graph of  $y = 2x - 3$ .  
 c What are the coordinates of the point where the graph crosses the  $y$ -axis?



**Worked example**



**PROGRESS BAR** Colour in the progress bar as you get questions correct. Then fill in the progression chart on pages 111–113.

1 Write down the next term in each of these sequences.

- a 10, 15, 20, 25, .....      b 25, 22, 19, 16, .....      c 16, 8, 4, 2, .....

2 For the sequence 30, 26, 22, 18, ... write down

- a the term-to-term rule .....  
 b the next three terms. ....

3 A sculptor uses metal pieces to make a wall decoration. It grows like this.



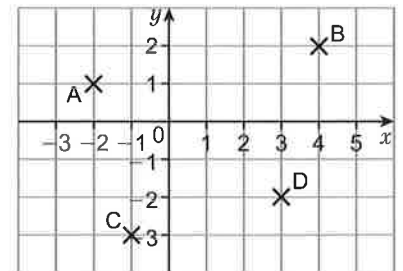
a Complete this table.

Shape number	1	2	3	4	5
Number of metal pieces	3	5			

b Describe the term-to-term rule of the sequence. ....

4 Write down the coordinates of points A, B, C and D.

- A .....  
 B .....  
 C .....  
 D .....



5 a Complete this table of values for the graph of  $y = 2x + 10$ .

$x$	1	2	3	4	5
$y$	12				

b What is the  $y$ -coordinate when the  $x$ -coordinate is 10? .....

6 Write the first four terms of the sequence whose  $n$ th term is

- a  $10n + 1$  .....  
 b  $n^2 + 2$ . ....

7 a Complete the table for values of the graph of  $y = 2x$ . Draw and label the graph.

$x$	0	1	2	3
$y$				

b Complete the table for values of the graph of  $y = -x$ . Draw and label the graph.

$x$	0	1	2	3
$y$				

